2.12. Neutrosophic Topologies.

A) General Definition of NT:

Let M be a non-empty set.

Let $x(T_A, I_A, F_A) \in A$ with $x(T_B, I_B, F_B) \in B$ be in the neutrosophic set/logic M, where A and B are subsets of M. Then (see Section 2.9.1 about N-norms / N-conorms and examples):

$A \cup B = \{x \in M, x(T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B)\}$,

$A \cap B = \{x \in M, x(T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)\}$,

$\mathcal{C}(A) = \{x \in M, x(F_A, I_A, T_A)\}$.

A General Neutrosophic Topology on the non-empty set M is a family η of Neutrosophic Sets in M satisfying the following axioms:

- $\mathbf{0}(0,0,1)$ and $\mathbf{1}(1,0,0) \in \eta$;
- If A, B $\in \eta$, then $A \cap B \in \eta$;
- If the family $\{A_k, k \in K\} \subset \eta$, then $\bigcup_{k \in K} A_k \in \eta$.

B) An alternative version of NT

-We cal also construct a Neutrosophic Topology on NT = $]^-0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set ∅, called open sets, which is closed under set union and finite intersection.

Let A, B be two such subsets. The union is defined as:

$A \cup B = A+B-A\cdot B$, and the intersection as: $A \cap B = A\cdot B$. The complement of A, $C(A) = \{1^+\}-A$, which is a closed set. {When a non-standard number occurs at an extremity of an internal, one can write "]" instead of "(" and "[" instead of ")".} The interval NT, endowed with this topology, forms a *neutrosophic topological space*.

In this example we have used the Algebraic Product N-norm/N-conorm. But other Neutrosophic Topologies can be defined by using various N-norm/N-conorm operators.

2.13. Neutrosophic Sigma-Algebra.

The collection of all standard or non-standard subsets of $]^-0, 1^+[$, constitute a *neutrosophic sigma-algebra* (or *neutrosophic* σ *-algebra*), because the set itself, the empty set ∅, the complements in the set of all members, and all countable unions of members belong to the -power set $P(]^-0, 1^+[)$.The complement of a subset is defined above.

The interval NT, endowed with this sigma-algebra, forms a *neutrosophic measurable space*.

2.14. Generalizations:

When the sets are reduced to an element only respectively, then

$t\_sup = t\_inf = t$, $i\_sup = i\_inf = i$, $f\_sup = f\_inf = f$,

and $n\_sup = n\_inf = n = t+i+f$.

Hence, the neutrosophic logic generalizes:

-the intuitionistic logic, which supports incomplete theories (for $0 < n < 1$, $0 \leq t, i, f \leq 1$);

-the fuzzy logic (for $n = 1$ and $i = 0$, and $0 \leq t, f \leq 1$); from "CRC Concise Encyclopedia of Mathematics", by Eric W. Weisstein, 1998, the fuzzy logic is "an extension of two-valued logic such that statements need not to be True or False, but may have a degree of truth between 0 and 1";

-the Boolean logic (for $n = 1$ and $i = 0$, with t, f either 0 or 1);

-the multi-valued logic (for $0 \leq t, i, f \leq 1$);

definition of <many-valued logic> from "The Cambridge Dictionary of Philosophy", general editor Robert Audi, 1995, p. 461: "propositions may take many values beyond simple truth and falsity, values functionally determined by the values of their components"; Lukasiewicz considered three values (1, 1/2, 0). Post considered m values, etc. But they varied in between 0 and 1 only. In the neutrosophic logic a proposition may take values even greater than 1 (in percentage greater than 100%) or less than 0.